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ABSTRACT

This report is divided into four sections. In the first section mathematics education in New York state is compared with that in other countries and other states, and variations within the state are discussed. Six major problems are identified for further study: the decreasing interest and achievement in mathematics; the relationship of socioeconomic status of parents to children's achievement; the waste of female talent in mathematics; recently lowered standards in teacher certification; the encouragement of creativity in schools; and the effect of the Regent's examinations. In the second section promising practices in mathematics education are summarized, including activity methods, discovery teaching, integration with other subjects, individualized instruction and the involvement of aprents and the community. Professional problems are discussed in the third section: accountability and performance contracting, certification, and the use of specialist teachers and para-professional assis ce. Recommendations regarding specific actions unde ings used in the previous sections are presented in the final section. (MM)

# MATHEMATICS EDUCATION IN NEW YORK STATE

September 7, 1971

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TABLE OF CONTENTS

# Introduction

The world in which we live is changing. One result of this change is that people are required to think more quantitatively, as citizens, as consumers, and in their vocations. As citizens we are constantly bombarded with statistics of all sorts, relating to taxes, government programs, umemployment, defense, education, environment, elections and so on. As consumers, we are exposed to statistics about the value or danger of the food we eat, the air we breathe, and the activities in which we participate. The jobs we hold now rely more heavily on quantitative thinking than at any time in the past. Not only are there many new positions which rely upon quantitative thinking (such as work with computers, systems management, biometrics, and data processing), many positions which formerly required little or no quantitative thinking now require a great deal. People in all aspects of business, education, medicine, construction, and other professions are finding that the amount of quantitative thinking required of them is constantly ar[illegible] increasing.

The world is changing. The schools are not changing rapidly enough. Perhaps the signs posted on the roads near schools which say "SLOW SCHOOL" do not constitute a grammatical error, but rather an unfortunately accurate description.

Available evidence suggests that mathematics education in New York State is no worse than that in other parts of the nation, and in many respects is probably better than average. Futhermore, similar evidence would support the argument that in many respects the United States is doing a better job of teaching mathematics to children than other countries. Closer examination of the evidence suggests that such a statement is merely damning these school systems with faint praise, however, since there is no evidence to suggest that the children of any state or nation are getting an adequate education in mathematics. Futhermore, there are disturbing pieces of evidence that suggest that the situation in New York State with respect to teacher certification and education, and students' interest and achievement in mathematics, has deteriorated recently.

The picture is not entirely gloomy, however, for we are entering a time in which the opportunities for improvement are as great as they have ever been. The teacher shortage is approaching an end in mathematics, as well as in other subjects, but we must end the shortage of truly qualified excellent teachers. Our knowledge of teaching and learning is increasing rapidly, and technology has supplied us with many new instruments which might be used to improve education. Now, we must learn to use our knowledge and these instruments to make education more effective, and more humane. The opportunity for great improvement exists. W must all accept the challenge.

In this paper, the state of mathematics education will be discussed as seen from an international, national, and state wide view point. Specific problems will be noted and examined. Promising practices will be discussed in light of research evidence. Recommendations will be made suggesting actions that the citizens of New York State can take to improve the quality of mathematics education in the State.

## STATUS OF MATHEMATICS EDUCATION

In order to evaluate the status of a project, some sort of comparison is necessary. The standard used may be some sort of absolute scale or it may be the condition of similar projects carreid out at different times or different places. In evaluating mathematics education, we shall begin by comparing mathematics education in the United States and in New York State with that of previous times and of other countries. Then, we shall consider the state of the art as it relates to the more absolute standard of our hopes and legitimate expectations.

Comparing the results of two different educational systems is difficult, and always of doubtful value. If the societies served by the systems differ in social attitudes, technological development, and ultimate goals, then the choice of tests to be used to evaluate the educational systems presents an essentially insurmountable problem. Futhermore, even if the attitude, technology, and goals are the same for the two societies, the ultimate value of the educational system should probably be determined by finding out what kind of life was lived by the products of the system, and how those lives were changed by the education received in the system. Clearly, this is impractical, first because the data is essentially impossible to collect and second because even if it were collected, it could not be available until long after it could be used to make any constructive changes. The comparisons which follow are the best available. However, they must be considered in the light of the severe limitations on any comparisions of this sort.

Historical. It is well known that the quantity of children educated has been increasing regularly throughout the history of formal education in the United States. This increase has been particularly evident in the twentieth century, and has continued right up to the present time. In New York State, for example, the number of children attending secondary schools (grades 9 through 12) increased from 740,000 1940 to 1, 171,000 in 1970. This change is partially accounted for by the increased number of children of school age (the number of students enrolled dropped to a low of 590,000 in 1952 because of population changes), but it is also true that the holding power of the high schools has increased steadily.

There is some evidence that the quality of mathematics education has also improved during the course of this century. In 1912, an international commission reported that approximately one-half of the high schools in this country offered only two years of mathematics (ninth grade algebra and tenth grade geometry), and only 20% of the schools offered more than two and one-half years of mathematics. Today, it is uncommon to find a high school that does not offer four years of mathematics. The 1912 commission went on to call attention to the fact that more than half of the teachers then teaching mathematics in the high schools had only one year or less mathematics beyond the mathematics course which they were teaching. Today, certified teachers generally have three to four years, or more, mathematics beyond the courses they teach.

Some disturbing developments, both with respect to teacher certification and student interest in mathematics within the State of New York will be discussed in a later section, but the long term historical view seems to suggest that mathematics education is in better shape today than it has generally been in the past.

International Comparisons. Research studies comparing the mathematics education of children in the United States with those from other countries have been relatively few, and often of doubtful validity. Until 1967, the results of these studies could be summarized as follows:

1. When children of country A are compared with children of country B on a test created in country A, the children of country A generally get better scores no matter what the two countries involved are.

2. The curriculum in the United States is generally "behind" that in other technologically advanced countries in the sense that a given topic (geometry, for example) is likely to be studied one or more years later in this country then in the other countries.

3. More time tends to be spent studying mathematics in the elementary schools of European countries than in the United States.

4. Attitudes of children in this country tend to be more positive towards mathematics than the attitudes of children in other countries.

5. The number of children continuing their education in mathematics into secondary schools is far greater in this country than in other countries when compared to the total number of children of secondary school age.

6. The mathematics studied in the secondary schools of other technologically developed countries tends to be more difficult and more advanced than that typically taught in this country. In England, it is also more closely related to the sciences and technology.

In 1967, the results of a major international study of mathematics education were reported. This study, edited by Torsten Husen, led to many reports in the news media that suggested that the teaching of mathematics in this country is not as good as that of other countries in the study. In fact, the evidence from the study itself does not suggest this conclusion. The authors of the study advise readers not to make international comparisons on the basis of the test scores, because it is clear that the tests are more oriented to the curricula of some countries than others. For the eighth grade test, the country in which children had the greatest opportunity to learn the subject matter of the test was Japan and the country in which children had the least opportunity to learn the subject matter of the test was Sweden.

The test scores ranked Japan the highest on this test and Sweden the lowest. The United States was second from the bottom both in opportunity to learn the content of the test and in average score on the test. Thus, these test results simply reinforce the previously known fact that many mathematical subjects are studied earlier in the schools of other countries than in the schools of this country.

At the present time, there is no clear research evidence that tells us whether it is desirable to teach more mathematics to children at an earlier age. The famous Swiss Psychologist, Jean Piaget believes that children must go through certain stages of learning and that to try to accelerate these stages is likely to result in "brittle" learning which does not stand up under stress. There is a great deal of evidence, however, that tells us that is is possible to teach more mathematics to children much earlier than we now do. Whether such acceleration will be beneficial or harmful in the long run is not now known for any particular subject, though there is evidence that training in some things too early is actively harmful. Continued evaluation and re-evaluation of our mathematics curriculum is clearly needed.

Other statistics from the international study are also of interest. The percentages of children in secondary schools and the percentages taking mathematics are of particular interest. In the United States, 70% of the children who are the right age to be seniors in high school are in school. In Japan, the figure is 57%, and in Sweden and Australia it is 23%. In all other countries of the study, the figure is less than 20%. The percent of children who are the right age to be seniors in high school and are actually taking senior mathematics is 18% for the United States, 16% for Sweden, 14% for Australia, 8% for Japan, 7% for Finland, and 5% or less for each of the other countries in the study. These percentages suggest that, at least in quantity, the United States is doing a good job. They also suggest that any comparison using all students taking mathematics in the senior year of high school would produce spurious results. Clearly, if the fastest 18% of the children in the United States were asked to run a mile and the fastest 5% from England were asked to run the mile, and then the average speeds of the two groups were compared, one would expect the average speed of the English 5% to be better than the average speed of the American 18%. Thus any such comparison is essentially meaningless.

Instead, the way to compare the quality of the mathematical knowledge of the best seniors is to look only at the corresponding percentages from each country. Since each country in the study has at least four percent of its senior aged children studying mathematics, such a comparison would be appropriate if the test were appropriate for the goals of all the countries. When such a comparison is made, the differences between the countries becomes negligible with only Sweden and Japan looking considerably better than the other countries.

When the really elite mathematics students of the various countries are considered, the United States looks particularly good. Students scoring in the top 1% on the test represented only three countries. Forty-six percent of these students were from the United States, 41% were from Sweden and 13% were from England. When the top five percent of those taking the test are considered, the United States ranks third, and for various other cut-off points the United States is generally better than the median. The numbers represented here are small so the statistical reliability of these data is doubtful, and the validity of the tests for the goals of the various countries can still be questioned.

Thus these data do not support the contention that mathematics education in the

United States is better than that in other countries any more than they support the contention that it is worse.

Presumably, one of the ways to evaluate mathematics education is to determine what the children think of mathematics. If they like it and wish to take more of it, the schools have presumably been at least partially successful, since it is the long term knowledge of, and use of, mathematics in which we are truly interested. In the international study, interest in mathematics and desire to take more of it were measured, and the children in the United States ranked higher on these measures than the corresponding children from the other countries in the study. Again, the evidence suggests that the United States is not doing a worse job in mathematics education than other countries, and perhaps is doing a better job.

There are two particularly significant pieces of information reported in the international study which are not primarily of an inter-country comparison nature. The first of these is that the status and education of parents is a very good predictor of the child's probability of being in an advanced mathematics class, and is also a good predictor of how well he will do in that class. Since this variable is different for different countries, it would appear that "inherent ability" can not be the entire explanation of this correlation, and it seems probable that society's estimate of a child's worth (determined by his parents' ability to succeed in the system) is a substantial factor in determining the child's subsequent success in school. Although this correlation is lowest of all the countries in the study in the United States, (and highest in Germany), there undoubtedly is considerable room for improvement on this score in the United States.

The second piece of information which seems to be of particular significance has to do with school starting age. When taken as a total group, children who started school at the age of six did better on the tests at the age of thirteen than children who started at either the age of five or the age of seven. Seven year old starters were next best. However, when the children were also divided according to socio-economic status, middle class children did better the earlier they started school, and lower class children did worse the earlier they started school This suggests that the schools of all nations are designed primarily to accomodate the middle class children, and that not only do they apparently not help the children from lower socio-economic backgrounds, they appear to actually damage them. There is substantial anecdotal evidence to support this contention, but research evidence on the subject is not substantial.

In summary, evidence from international studies makes it clear that mathematical subjects are taught later in the schools of this country than in most other "developed" countries, but does not tell whether this is good or bad. The United States is apparently not doing worse than most other countries in terms of student achievement and appears to be fostering better attitudes and keeping more children in mathematics classes than other countries. No countries appear to be doing a good job of educating the non-middle class children in mathematics, but the United States appears not to be as bad at this as other technologically developed countries.

Comparisons Between States. Information comparing mathematics achievement between states is sparce, and most available data pertains only to achievement of the academically talented. The available information regarding the academically talented suggests that such students are doing better in New York State than is generally the case over the nation as a whole.

In the National Merit Scholarship Qualifying tests, the New York State cut-off scores are among the highest in the nation. For 1970, New York State, Connecticut, and Delaware tied for the highest scores, and over the years, New York State has consistently been among the highest.

On advanced placement tests of the College Entrance Examination Board, New York State has had between 23% and 26% of the test takers each year since 1959, even though the state has only 9% of the population. Scores in mathematics have been approximately the same for New York State as for the rest of the country.

The Westinghouse Science Talent Search includes both science and mathematics, and therefore gives some indication of the quantitative quality of the high school students in the various states. Since 1942, the percent of winners and honorable mentions in New York State has fallen below 20% in only one year (1943 with19.7%) and has been as high as 41.7% (in 1953). Over the entire period, New York State has won about 30% of the awards.

What little evidence that is available suggests that New York State is doing a good job of educating the academically talented in mathematics when compared with the rest of the United States.

<u>The Situation Within New York State</u>. While New York State appears to have done an average or better than average job of educating children in mathematics, there remain several serious problems which exist on a national and international level, and which are also common to New York. Among these are the education of non-middle class children, the place of women in mathematics, teacher education and certification, and an educational structure which apparently does not foster creativity on the part of either teachers or students. Beyond these problems, there are certain disturbing trends that have appeared in the last few years in New York State which may signify even more serious

trouble ahead. These include the amazing change in certification requirements that has recently occurred, and the apparent downswing in interest in mathematics on the part of high school students.

Decline In Interest and Achievement. One example of the apparent reduction in interest in mathematics among New York State school children can be seen in the numbers of students taking Regents Examinations in advanced mathematics courses over the past thirty years. When this figure is compared with the total number of pupils in grades nine through twelve, it is particularly revealing.

TABLE I

Number of students writing Regents papers in tenth, eleventh, and twelfth grade mathematics courses from 1940 through 1969; and the percent this number is of the total number of students in grades nine*, ten, eleven and twelve.

| YEAR | NUMBER | PERCENT | YEAR | NUMBER | PERCENT |
|---|---|---|---|---|---|
| 1940 | 162,045 | 21.4 | 1955 | 164,623 | 26.5 |
| 1941 | 163,850 | 21.3 | 1956 | 179,612 | 27.7 |
| 1942 | 163,412 | 22.8 | 1957 | 200,394 | 28.7 |
| 1943 | 155,500 | 22.7 | 1958 | 230,866 | 30.7 |
| 1944 | 151,923 | 24.0 | 1959 | 260,215 | 32.8 |
| 1945 | 157,642 | 25.0 | 1960 | 263,697 | 32.4 |
| 1946 | 179,884 | 28.3 | 1961 | 259,275 | 30.4 |
| 1947 | 186,284 | 29.1 | 1962 | 280,748 | 31.1 |
| 1948 | 173,650 | 27.7 | 1963 | 307,862 | 32.1 |
| 1949 | 160,830 | 26.4 | 1964 | 271,575 | 26.8 |
| 1950 | 153,911 | 25.7 | 1965 | 272,662 | 26.2 |
| 1951 | 150,546 | 25.5 | 1966 | 272,871 | 25.7 |
| 1952 | 149,211 | 25.2 | 1967 | 271,669 | 25.0 |
| 1953 | 145,052 | 24.3 | 1968 | 246,970 | 22.1 |
| 1954 | 151,697 | 25.3 | 1969 | 235,335 | 20.4 |

*The available total includes the 9th grade. If the assumption is made that approximately one-third of the students are in the ninth grade, the figures in the percent columns should be multiplied by four-thirds. Thus, the first percent would be 28.5 instead of 21.4 and the last would be 27.2 instead of 20.4. The trends indicated would be the same, however.

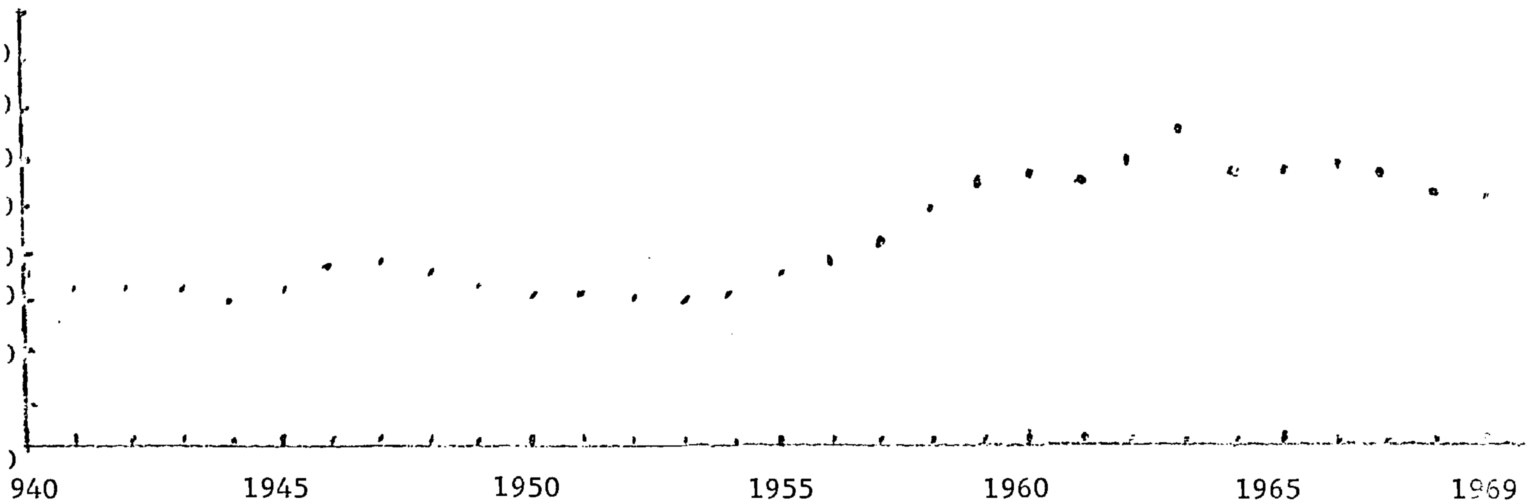


FIGURE 1. Number of students writing Regents papers in tenth, eleventh, and twelfth grade mathematics courses, from 1940 through 1969.

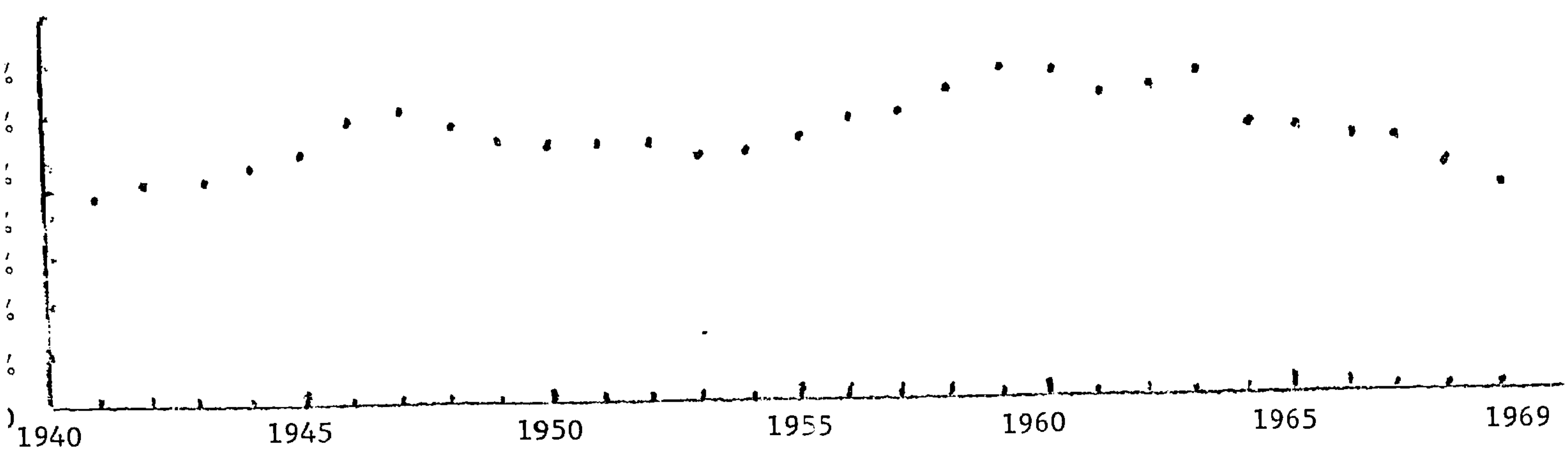


FIGURE 2. Number of students writing Regents papers in tenth, eleventh, and twelfth grade mathematics courses as a percent of the total number of students in <u>ninth</u>, tenth, eleventh and twelfth grades from 1940 through 1969.

An examination of Table I, and Figures 1 and 2 reveals the fact that while secondary school enrollment has been rising steadily since 1952, the percent of students taking advanced mathematics courses in New York State stopped rising in 1959 and has been falling steadily since 1963. Futhermore, even the absolute numbers of students taking mathematics Regents examinations have been getting smaller in recent years. In a society that is becoming ever more quantitative, and which will become more dependent on quantitatively trained people to solve future problems in the social sciences as well as in the natural sciences and business, this trend is most disturbing.

There is also evidence supporting the hypothesis that New York State's excellent position with respect to the rest of the country in mathematics education may be slipping. For example, the Mathematics Student Journal is a nationally distributed mathematical journal of interest to high school students of mathematics. During the early and middle 1960's, students sending in answers to the problem section of the Mathematics Student Journal from New York State constituted about 25% of the students solving problems from the entire nation. During the late 1960's this figure dropped to about 20%, and it now appears to be less than 15 percent.

In a similar manner, the figures for the Westinghouse Talent Search, while suggesting that New York State's position in absolute terms is better than average, suggest that its relative position is slipping. Since 1942, New York State has had about 30% of the winners and honorable mentions, but in the past six years has produced less than 24% of the winners and honorable mentions. A similar drop in the percent of students from New York State taking advanced placement tests since 1964 appears to exist, though long term figures don't exist becuase of the newness of the program (1957 was the first year that New York was represented in the program).

Pupil Evaluation Program. Interesting data about mathematics achievement within New York State elementary schools is available through the Pupil Evaluation Program. The Pupil Evaluation Program (PEP) was created in 1965 to provide information for efficient management of ESEA, Title I funds. It collects and analyzes objective data on achievement in reading and mathematics in grades 3 and 6. There is also a "minimal competency test" which is administered to many ninth grade pupils in the state.

During the first year of its operation, minimal competency was defined by how well pupils did with respect to other pupils in New York State. Subsequently, State Department of Education Specialists were asked to review the tests and indicate how many items a pupil would have to miss before the results would suggest the need for compensatory help beyond that normally available in the classroom. This figure was then adjusted to correspond to the twenty-third percentile of the 1966 distribution of scores for each test. Given this procedure, it is clear that the fact that 23% of the children who took the tests in 1966 fell below the minimal competency score is neither surprising, nor an indication that something is wrong with the schools, unless the evaluation of the test items by the State Department of Education Specialists is accepted as valid. In any case, the test scores do not provide data with which to compare New York State with other states.

In mathematics, the tests were changed in 1969 to reflect recent major changes in the mathematics curriculum, but scores were pegged to the old 1966 standards to permit comparison. A review of the mathematics tests now being used makes it clear that the tests do test concepts, computational ability, and problem solving ability that would be considered to be important by most mathematics educators and laymen.

While the PEP scores do not offer a reasonable way to compare New York State with other states, they do provide interesting information which can be used to help identify problem areas within the State, and also provide evidence of certain trends. In Tables II and III, schools are categorized into seven types: Type I includes all cities with populations greater than 1,000,000 (New York City is the only city in this category), Type II includes cities between 100,000 and 1,000,000 population, Type III includes cities with populations between 50,000 and 100,000, Type IV includes cities with populations less than 50,000, Type V includes villages and large central districts with enrollments greater than 2,500, Type VI includes rural districts with enrollments between 1,100 and 2,500, and Type VII includes rural districts with enrollments less than 1,100. The median percentile rank for the entire state in 1966 was, by definition, 50. Ranks higher than 50 suggest scores better than the state wide figures for 1966 and ranks lower than 50 suggest scores below those for the entire state in 1966.

TABLE II

Median Percentile Ranks in Grade 3, Compared to Statewide Figures for 1966, for PEP Mathematics Tests.

| Type | 1966 | 1967 | 1968 | 1969 | 1970 |
|---|---|---|---|---|---|
| I | 29 | 31 | 37 | 37 | 38 |
| II | 46 | 44 | 43 | 48 | 45 |
| III | 55 | 54 | 54 | 56 | 54 |
| IV | 57 | 56 | 58 | 58 | 57 |
| V | 63 | 63 | 63 | 65 | 63 |
| VI | 59 | 60 | 60 | 60 | 59 |
| VII | 57 | 57 | 57 | 58 | 56 |
| State wide | 50 | 51 | 53 | 54 | 53 |

TABLE III

Median Percentile Ranks in Grade 6, Compared to Statewide Figures for 1966, for PEP Mathematics Tests.

| TYPE | 1966 | 1967 | 1968 | 1969 | 1970 |
|---|---|---|---|---|---|
| I | 34 | 30 | 32 | 30 | 28 |
| II | 49 | 48 | 46 | 39 | 37 |
| III | 51 | 50 | 48 | 46 | 43 |
| IV | 54 | 52 | 51 | 48 | 46 |
| V | 58 | 56 | 54 | 54 | 51 |
| VI | 59 | 54 | 53 | 51 | 50 |
| VII | 59 | 56 | 54 | 52 | 50 |
| State wide | 50 | 47 | 47 | 44 | 42 |

New York City (Type I) stands out as the school system in which the children are apparently in need of the most help. The low scores in New York City result in 45% of the third grade public school children, and 51% of the sixth grade public children being classified as educationally disadvantaged in mathematics, as defined by the specialists from the State Education Department.

There are many possible reasons for the fact that New York City scores are lower than those for the state in general. Jack A. Maybee, Chief of the Bureau of Pupil Testing and Advisory Services, points this out when he says:

> The achievement of a single pupil or of all the pupils in a school, a community, or the State is a result of the interaction of at least three types of factors:
>
> Educational Resources - the total environment in which the school or school system is located, including community aspirations, financial support, and other socioeconomic conditions,
>
> Teaching and Learning Setting - the appropriateness and quality of instruction, curriculum, supervision, organization, and other educational services provided by the school or school system,
>
> Pupil Potential - the physical, emotional, social, and mental characteristics of the pupil, including motivation, interests, readiness. attitudes, and abilities.

Low test scores, therefore, cannot reasonably be used to criticize teachers, administrators, or others associated with a school or school system. They CAN, however, be used to indicate where great need exists for educational help. In this context, it seems clear that New York City is in need of substantial help. This help could take the form of specialists to work with children who are in need of help, or it could take various other forms.

Trends over the five years in which the tests have been given are also of interest. For the third grade test, the State wide average has risen from 50 to 53. This is attributable to an impressive improvement in scores in New York City, since the scores for the rest of the State have remained constant or been reduced over [illegible] period. The situation in mathematics in the primary grades of New York City appears to be improving, perhaps as a result of the extra aid assigned there as a result of the PEP scores. In the sixth grade, the situation does not look as promising. Scores on the test throughout the State have become substantially worse in all types of schools.

The analysis of data within New York State seems to suggest that interest and achievement in mathematics in the State are both declining. There are many possible reasons for this decline. It may be a result of a general nation wide trend towards anti-intellectualism. It may be a result of recent temporary changes in the job market. It may be the result of something going on within the schools. Whatever it is, however, there seems to be little question of a long range need for a quantitatively educated citizenry, and a large number of highly educated scientists, operations research specialists, and others whose professions are primarily dependent on quantitative ability and education.

Teacher Certification. Given the obvious problems in mathematics education in the State, and predictions of reputable groups (such as the Panel on Teacher Education of the Committee to Survey the State of the Mathematical Sciences of the Conference Board of the Mathematical Sciences) that the teacher shortage in mathematics is rapidly drawing to a close, it is most surprising that the Division of Teacher Education and Certification of the State of New York should see fit to drastically reduce the mathematical requirements needed to become a teacher of mathematics.

In September, 1969, it became possible for a teacher in New York State to be certified to teach mathematics in secondary schools with only 18 hours of mathematics taken in a college. There is no need for the 18 semester hours to include any calculus, even though there are hundreds of calculus courses being taught in the high schools of the State, and the teacher might end up teaching one of them. In fact, if the recommendations of the prominent Cambridge Conference on School Mathematics are followed, much of what is now in a typical college calculus course will be taught to junior high school children before teachers now being certified are old enough to retire.

The 1969 certification regulations require teachers of English, social studies, and science to have 36 hours in their speciality; and teachers of foreign language are required to have 24 semester hours. Prior to the 1969 changes, teachers of mathematics were required to have 33 semester hours including at least one full year of differential and integral calculus. Futhermore, at least 15 semester hours of that total had to be in advanced courses (beyond the calculus, and not generally taught in the high schools). Today, it would be possible for a prospective teacher to be certified to teach mathematics in a school in which many of the seniors would have had more advanced courses than the teacher. Furthermore, if the Cambridge Conference recommendations are even approximately met, a teacher who is certified to teach mathematics today could, before he retires, easily find himself in the position of having had less formal advanced mathematical education than typical junior high school pupils.

Most of the State's teacher preparatory schools are holding to more substantial requirements in mathematics than those required by the State, but this will not prevent large numbers of substandard mathematics teachers from being certified without the blessing of a New York State teacher preparatory institution. If this policy is allowed to continue for the next five or six years, it can be predicted that the end of the shortage in available mathematics teachers will approximately coincide with the filling, on permanent tenure, of a majority of the State's teaching positions in mathematics by teachers with inadequate mathematical backgrounds. For example, the number of certificates issued to teach mathematics increased by more than 11% between 1969 and 1970 (from 2734 to 3040) while the total number of teaching certificates issued in the State increased only about 5% (63,182 to 66,417).

The state of affairs regarding certification of elementary school teachers is no better. In order to receive certification to teach elementary school, a person need not take a single course in mathematics. The most prominent professional mathematics group that has addressed itself to the certification of elementary school teachers (the Commission on the Undergraduate Program in Mathematics, Teacher Training Panel, of the Mathematical Association of America) has recommended that elementary school teachers be required to take 12 semester hours of mathematics.

Available evidence makes it clear that the average requirement by teacher training institutions over the country is approximately six semester hours of mathematics courses for a prospective elementary school teacher, which is a substantial upward change (of about three points) from the time ten years ago when the CUPM recommendations were originally made.

Problems. While New York State's position in mathematics education with respect to the rest of the nation and the world appears to be superior at this time in history, there is strong evidence to support the contention that there are serious problems, some of which may be unique to New York State and some of which are quite general. Some of the most pressing of these problems have already been identified and will simply be reiterated here. Others will be discussed in somewhat more detail.

1. Decreasing interest in mathematics and decreasing achievement in mathematics seems to have been documented both on an absolute scale and in relation to the rest of the nation.

2. The relationship of socioeconomic status of parents to children's achievement is a serious problem. Available evidence suggests that the United States is doing a better job of solving this problem than other advanced technological nations, but the problem still exists. Mathematics appears to be the subject matter area in which it is most possible for children from Spanish speaking homes and Negroes to succeed, from the research evidence that is available. Probably, this is a result of the fact that most other school subjects are more highly dependent on the use of standard English found in middle class, Caucasian, English speaking homes. The symbolism of mathematics apparently makes it an ideal vehicle to reach many of the unreached, but talented, children in our society. Since the PEP scores locate the major problems in elementary education in the major cities of the State,

it would appear that sufficient efforts are still not being made to reach the non-middle class, non-white, non-English speaking children of our community, since it is known that they are generally located in the large cities. The recent upturn in third grade mathematics scores in New York City is the one bright spot in this otherwise gloomy picture.

3. A problem which is not unique to New York State, but is common to [illegible] to do with the waste of female talent in mathematics. In elementary schools, boys and girls have about equal talent and interest in mathematics. When they reach junior high school, girls begin to lose interest and show signs of reduced achievement. There is reason to believe that social pressures make girls believe that they OUGHT NOT to be interested in mathematics and sciences, and therefore they shy away from the subjects. As prejudices of this sort have begun to fall, some women have found their way into the mathematical sciences and natural sciences and demonstrated that women can be successful in quantitative pursuits, but it is still true that only 7% of the Ph.D.'s awarded in the mathematical sciences last year were awarded to women. This statistic indicates one of the great wastes of quantitative talent in the United States (and most of the rest of the world) today.

4. The teacher certification problem has already been documented. At a time when the teacher shortage in mathematics education appears to be drawing to a close, New York State, rather than raising its standards to take advantage of this opportunity has substantially lowered its standards for secondary certification, and has failed to keep pace with the rest of the nation by requiring some mathematics education for its elementary school teachers.

5. Another problem that has plagued educators for years has to do with creativity. The difficulty in collecting hard data on this subject is largely

a result of our inability to design a test which can clearly be said to measure creativity. In spite of the lack of hard statistical evidence, most experts on creativity seem to be agreed that children are far more creative when they enter school than they are when they leave it. Futhermore, the decrease in creativity seems to be a continuous process. Perhaps decreasing creativity is a price we pay for the maturation of our other physical and mental processes, but there are many examples of highly creative people who have matured in other respects. Perhaps some creative people find the unsupportive atmosphere of the school an excellent stone against which to whet their creativity, but anecdotal evidence suggests that far more children find their creativity dulled against that stone than sharpened by it. Virtually every good teacher can cite examples of children who had been given up as lost by the system, but who were saved to go on and make a positive contribution in the world through some act (often a chance action) of a teacher or other adult. There is good reason to believe that a good deal of our potentially most creative talent is being wasted, and that changes in the schools could prevent at least some of that waste. But the changes are not forthcoming.

To try to assess the blame for a lack of conditions in the schools that would foster creativity is a pointless exercise in semantics. The teachers can claim that if they try to "open up" their classrooms to encourage individualism and creativity the administration will complain. The administrators can point out that if they allow such apparent chaos in the schools the parents will complain. The parents, quite justifiably, can claim that colleges, businesses that hire young men and women, and other symbols of the establishment are not looking for creativity in school graduates and are most unlikely to reward the high school graduate who is creative and independent of mind -- quite the contrary, such institutions almost invariably penalize any substantial demonstration of creativity and independence. Perhaps our society does not really want to foster creativity and independence. If that be the case, then perhaps the schools are doing the job they are asked to do.

6. No discussion of creativity in the schools would be complete without some mention of the Regent's examination. Any person who has spent substantial time in the mathematics classrooms of New York State can cite examples of teachers who begin reviewing for Regents exams on the second day of class in September, of teachers who would like to follow the lead of an interested and excited student into some fascinating mathematical topic, but who dare not do so because the topic will not appear on the next Regents exam; of teachers who stunt the growth of talented students with whom they have been entrusted, so the students can spend more time preparing for Regents examinations and thus score 95 rather than 90; and many other examples of gross abuse of children in the name of higher Regents examination scores.

Regents examinations are not, however, in and of themselves evil. The examinations were originally created to assure minimal competency in students who were planning on attending college. There is substantial anecdotal evidence to support the contention that the worst college preparatory mathematics classes in New York State are not as bad as those in most other states, probably because the teachers in New York State at least teach their students what is going to be on the Regents. Further, it can be shown that the Regents examinations are the best predictors of success in college, better than the College Entrance Examinations which are designed for that purpose.

Before doing away with the Regents exams, or greatly modifying them, the citizens of New York State should decide what they really want. If the examinations are to be primarily a status symbol ("My John got a 97 on his mathematics Regents."), then they should continue as they are, and the present abuses will also continue. If they are to be used largely as a predictor of college success, they should probably not be greatly changed, assuming we are willing to accept the possibility that college success is an indication of a certain lack of creativity and independence of mind. If the Regents examinations are supposed to help maintain minimal educational quality and identify problem areas in which extra work is needed, perhaps they should be modified to more closely approximate the testing, scoring, and reporting procedures followed for the PEP scores. Even this procedure has the danger that it may foster a State wide prescribed curriculum, but it might be possible to reduce this danger by encouraging teachers and school administrators to identify innovative activities and invite State Department of Education personnel to visit these innovations, analyze them, and perhaps publicize them.

## Promising Practices in Mathematics Education

Education is a complicated business. Perhaps it is the most complicated of human endeavors, since it is an attempt to change the human mind on a massive scale, and the human mind is undoubtedly the most complex mechnaism in the physical world. Because of this complexity, it is essentially impossible to say that practice A is better than practice B for all situations. We can cite numerous examples of teachers who, by behaving in apparently incompetent ways, have produced outstanding students, because the students were forced to learn on their own. We can also cite numerous examples of educational research that suggested that a certain practice was the answer to all educational problems, along with subsequent research that showed that the indicated practice works only in very small doses when it is new to both pupil and teacher. In fact, all educational researchers are familiar with something known as the Hawthorne Effect, which, in slightly simplified form says that whenever a change is made in the conditions of human activity, and humans involved have reason to believe the change was made with their welfare in mind, they will respond more favorably to the new conditions. The Hawthorne Effect is so strong that it will work as changes are made from condition A to condition B, from condition B to condition C, and from condition C back to condition A. Thus, a naive researcher might conclude that A is better than B, B is better than C, AND C is better than A, from which it can easily be deduced that A is better than A.

The Hawthorne Effect, and other problems in educational research make it difficult, if not impossible, to say that one practice is best for all circumstances; however, there is research evidence, and there is a strong body of professional opinion to support the contention that certain practices in mathematics education are promising. Teachers, administrators, and citizens should be aware of some of these practices, and should be aware of what is known about them.

Activity Approach. Recently, there has been a great deal of discussion among mathematics teachers in this country about the activity approach to teaching mathematics, about mathematics laboratories, about games as an aid to teaching mathematics and about the British open school approach to teaching mathematics. Interestingly enough, virtually all of the available research on an activity approach to the teaching of mathematics comes from an earlier era in this country when the Progressive Education movement was in full swing. The research from that time suggests that learning mathematics through various activities which are meaningful to the child can be an effective means of teaching mathematics, but that an undirected, opportunistic approach to the teaching of mathematics is likely to be a failure. Indeed, the history of the gressive Education movement suggests that it went out of style largely because reme advocates of a child centered approach appeared to be advocating a practice that would teach the child nothing unless he expressed an interest in learning it. The difficulty, inherent in children who do not really know what they want to learn, is compounded in a sequential subject such as mathematics in which it is difficult to learn a given portion of the subject until a previous portion has been mastered.

If a reasonably well structured curriculum is followed, however, available evidence suggests that children develop better attitudes, and learn and retain mathematics better if it is taught as a skill that has practical applications outside of the classroom. Manipulative materials in general seem to be more effective in teaching children (especially elementary school children) than pictures, and pictures are more effective than no aids at all. Games, in general, seem to be an effective means of instilling interest and improving skills.

For the vast majority of children who study mathematics, it will be of value in direct proportion to the amount of use they are able to make of it in coping with the world around them. Therefore, it seems only reasonable that they should learn mathematics in relation to problems that are real to them. Studies relating particularly to the urban child suggest that the children use mathematics in their daily living in connection with time, money, counting, reading numerals, simple operations with whole numbers (subtraction, addition, and multilication, in that order, with very little use of division), measurements, and games. Presumably, it would be desirable for the schools to concentrate more heavily on these practical aspects of mathematics, while not emphasizing them to the extent of destroying the internal structure of mathematics. Thus, an ideal curriculum might be one in which each new topic is developed in connection with some aspect of the child's reality, than abstracted to show the general nature of the subject, and then practiced by the child in connection with some interesting application, such as a game or other motivational activity.

In connection with the activity approach, many very expensive, factory made materials have been created and sold to the schools. As of now, there is very little evidence supporting the contention that such expensive materials are of any great value. In fact, there is some reason to believe that such "home made" materials as popsicle sticks and rubber bands are as good as any aid in teaching children to understand their numeration system and the basic operations with whole numbers. Research on the highly publicized Cuisenaire rods seems to suggest that they are effective with certain children under certain circumstances, but are no more effective than no aids at all in other circumstances. It appears that individual teachers should be encouraged to use those aids which they find to be most effective in their particular situations.

Discovery Techniques. There are many arguments and substantial research supporting or condemning "the discovery approach" to the teaching of mathematics. There appears, however, to be no generally accepted definition of what the discovery approach is. In other disciplines, something referred to as the "inquiry (or enquiry) approach" appears to have similar characteristics.

One technique that appears to epitomize "discovery" for some people has the teacher ask a series of leading questions which the children answer either "yes" or "no", or possibly with some other short answer. This technique is exemplified in some of Plato's writings on the teaching of Socrates. The technique lends itself quite well to being adapted for programmed learning or computer assisted instruction. Experience of many teachers with this technique has led them to the conclusion that while children may be able to answer each question correctly as it is asked, they are quite often unable to reconstruct the reasoning of the entire process by themselves, and may be unable to recall even a portion of what they presumably have learned. The students quite often report having the feeling of being trapped by this discovery technique, and while they may admire the teacher's skill, they apparently sometimes resent the fact that they are not sufficiently skilled debators to win some of the classroom discussions. It is also quite often true that in a typical classroom setting, one or two of the students do most of the "discovering" and the others listen as they would to a lecture.

In another form of discovery, the teacher presents children with a situation and encourages them to draw conclusions about it. As practiced in the elementary schools, this usually involves the use of concrete objects, and the children quite often are using a form of scientific induction to arrive at their answers. In these cases,"discovery" and "scientific inquiry" appear to be virtually synonymous. As the children become more mature mathematically, they are encouraged to arrive at their conclusions by deduction as well as induction, and to try to convince others that they indeed have the right answer. This process seems much more closely akin to what mathematicians do when they are behaving like mathematicians,and would have the philosophical support of a large portion of the mathematical community. There is some evidence that a discovery approach such as this does NOT teach children in such a way that they do better on computational tests administered shortly after the instruction, but that the children do appear to retain their knowledge longer and transfer it to other situations more easily.

Probably the clearest information available with relation to discovery methods is that a teacher who can teach well with any pupil centered technique can also do a good job of teaching with a teacher centered technique, but the reverse statement is not true. Thus, a good teacher is probably able to use a discovery technique, and will use it when he believes it is appropriate to the students and subject matter to be taught.

Integration of Mathematics with other Subjects. Studies have shown that mathematical concepts appear in other subjects with very high incidence, and that children quite often do not understand the concept when it appears in other subject areas. The association of mathematics with the natural sciences is, of course, well known. It is also true, however, that the concepts of time, measurement, money, and distance are particularly heavy in the social sciences and in English. In one study, researchers found an average of 40 mathematical concepts per page of a social studies book, and similar results have been found for other social studies books and for English books. Futher, after the concepts were identified, the students were tested to see how many of the concepts they understood, and it was found that only 50% of the mathematical concepts involved in the texts were understood by the pupils using the texts. Clearly, there is great need to coordinate the study of mathematics with the study of other subjects.

In a sense, the elementary school classroom is the ideal place to coordinate these different subjects, since the same teacher is teaching them all. However, even in the elementary school, teachers generally seem to carry over little of the experience they have had in teaching one subject to the learning of the next. In the secondary schools, with departmentalization, the integration of subject matter from the various disciplines is, in general, even less. There are instances of informal cooperation between mathematics teachers and social science teachers, English teachers and natural science teachers, but the instances are all too few. The opportunities for team teaching across disciplinary lines in the secondary school are great and should probably be exploited. Projects on the environment, local budgets, city planning, race relations, etc., could easily involve all of the disciplines the students are studying, and rather than interfering with the regular classroom work could be expected to make it more meaningful.

Individualization of Instruction. Everybody appears to be in favor of individualization of instruction. Unfortunately, not everybody agrees what it is. Variations range from giving each child a book and letting him learn at his own rate (with the teacher as a sort of glorified record keeper, and occasional question answerer), to a situation in which the teacher lectures to an entire class and each student stops listening at the point where he chooses to stop listening. There are computer assisted instruction techniques which seem to make their contributions in the area of learning skills, and there are procedures in which one pupil teaches another pupil while the teacher is occupied with another part of the class. The activity approach already mentioned can be combined with individual analysis of children's problems by the teacher to produce yet a different form of individualization.

Research on computer assisted instruction, programmed instruction, and Individually Prescribed Instruction (IPI) have in general shown no favorable effects. In some cases the sponsors of such programs insist that even though standardized tests show no advantage to these systems, they somehow help to humanize the subject matter. There are, however, mathematics educators whose major objection to these techniques is that they dehumanize the subject. As of now, the only favorable research (by Suppes) suggests that the computer assisted instruction programs developed by Suppes to help primary children in the acquisition and maintenance of skills have been reasonably successful.

One of the more promising techniques mentioned under this heading is a procedure by which older children (juniors or seniors in high schools) are allowed to spend one period a day, several times a week, working with much younger children in the elementary school. The student tutors work on a one-to-one basis, teaching mathematics to the children. Results of such experiments have shown good progress on the part of the young pupils and also substantial improvement in both attitudes and achievement on the part of the student-tutors. Similar experiments in which children from the same age group have been used as tutors for their peers have not generally been as successful, as the mistakes of the tutors tend to be transmitted readily to the other children, and seem to be harder to correct for the tutee than for the child who originally produced the mistake.

A second promising procedure for individualizing instruction that is supported by research involves a combination of an activity approach, other relatively standard techniques with the entire class, and substantial individual diagnosis and prescription by the teacher or other highly trained adult. It is distinctly possible that at some time in the future computer procedures for diagnosing difficulties and keeping records will be sufficiently well developed, and sufficiently inexpensive, so that they will become a practical aid to the teacher. At present, however, the procedure requires substantial time, energy, and skill on the part of the teacher or other well trained adult.

An example of how this procedure works is the following: a child hands in a paper on which numerous problems of the following sort have been done incorrectly: $3 + \square = 5$, $\square + 2 = 7$, $4 + \square = 6$; while problems such as $4 + 5 = \square$, $8 + 1 = \square$ and $0 + \square = 7$ have apparently been worked correctly. The teacher, in many typical cases, will dutifully mark wrong all of those problems for which the incorrect answer appears, while praising the child for those answers that are correct. The child is confused, because he thinks he did all of the problems the same way. If the teacher were to analyze the situation with some care, he would very likely discover that the child has no idea of what the "=" sign means, and is simply adding any pair of numbers that appear in the same problem with "+" sign. If the student is expected to discover his difficulty by himself, he is at least as likely to decide that "+" sometimes means subtract as he is to hit on the correct solution. Once the teacher has correctly diagnosed the difficulty, he can prescribe various kinds of activities (activities with balance scales are good for this purpose; a game in which two children have numerals placed on their backs, are told the sum / the class and then are expected to guess the numeral on their back by looking at that on the other child's back is very effective, etc.) to remedy the situation.

Both the diagnosis and the prescription can be very difficult, and may require great skill. An example of this occurred in one New York State school recently, when a girl continually handed in papers of addition problems on which all her answers were incorrect except those in which the sum was supposed to be five or ten. The teacher had concluded that the girl simply had no idea of how to proceed in addition problems, but was intrigued by the fact that the girl usually got the correct answer when the sum was five or ten. A doctoral student who was working in the school had told the teachers that there is usually a reason for children's mistakes, so he was asked to analyze the paper and discover the reason in this case. Being unable to come up with a rational explanation, he handed the paper to his major professor. The professor was about ready to conclude that the child was simply putting down answers at random because somebody wanted her to put answers on the paper when he discovered the pattern in the answers and realized that what the girl was in fact doing was substantially more complicated than what she had been asked to do. Essentially, what the girl had done in each case is to find the compliment of the sum of the two numbers with respect to ten, rather than to simply write down the sum of the two numbers. In cases where the sum is ten, she apparently had discovered that writing "10" rather than "0" elicited a better response from the teacher. She also had made several mistakes within her own system -- an unsurprising fact, since children quite commonly make mistakes in arithmetic.

The moral of this story is that teachers should probably have outside help in diagnosing difficulties their pupils are having. Futhermore, it should be possible to set up a procedure by which specialists within a school system can go directly to professors of mathematics education, or other outside experts, for help with specific problems. Such a system should not take a large amount of professorial time, but should substantially improve the chances that truly creative children who have a hard time communicating with certain adults will be discovered, and helped over their problems.

Slow Teaching for Slow Learners. Another form of individualizing instruction that has been supported by recent research is based on the belief that every child can learn the essentials of mathematics, but that some children will learn those essentials more quickly than others. In this procedure, the children who are expected to learn more slowly are placed in special classes in which there is a great deal of practice, and support with manipulative materials and teacher contact. Research seems to indicate that such children can progress as far as other children if they are given such special attention and allowed to proceed more slowly . Taking the same course twice, however, does NOT produce these good results, and in fact tends to produce quite negative results. There are many problems connected with this procedure, not the least of which is the identification in advance of the child who is going to have trouble learning the subject at the usual rate. While it is easy to identify some such children, there will always be the questionable border line cases. If such a procedure is used, great flexibility in moving children from one class to another, with special tutoring services where needed, would be essential.

Involving Parents. In the past, and especially during the "revolution in school mathematics" it was common for the teachers to ask parents not to help children with their mathematics because the parents were likely to confuse the children. Research has shown, however, that increased knowledge of what is supposed to be happening in the school mathematics class on the part of the parent results in better achievement on the part of the pupils. Some teachers have always tried to make a point of involving parents through parent-teacher conferences, through letters home, and through activities which the children can take home and play with parents or siblings. There is good reason to believe that if teachers were to send letters home at regular intervals explaining to the parents what was about to happen, what their child was supposed to be learning and a few simple activities in which the parents or other members of the family could participate with the child to help him learn, the results would be beneficial. The letters would be mimeographed and sent home at the beginning of each unit or chapter. Presumably, such an activity would increase interest, knowledge, and participation on the part of parents -- even those parents who are unable to attend PTA meetings -- and thus, the activity could be expected to help raise children's achievement levels.

Involving the Community. In virtually every community, there are resources that could be used to benefit the mathematics education in the schools of the community. In some communities, there are practicing mathematicians or scientists, in many communities, there are practicing engineers or computer programmers, in most communities, there are business men who use some mathematics in their activities (bankers, grocery store owners, gas station operators, etc.). Available evidence suggests that when children see mathematics as something which has practical value, their attitudes improve towards it, and they presumably try harder to learn the subject. While it would be nearly impossible for an ordinary teacher to become aware of all of the possible resources available in or near his community, it would not be difficult for a team of teachers, working in the summers, to compile a list of community resources that are available to help in the mathematics classes at certain specified times. There would have to be some central control so that certain individuals were not abused by being asked to spend too much time, but most citizens of our communities would be happy (indeed, honored) to be asked to help with the process of education in the local schools. . . especially if the news media were encouraged to publicize such activities. Most such activities, including field trips, have been carried out on a relatively hit-and-miss basis in the past, but appear to have been somewhat successful.

The so-called SEED program, in which mathematicians and scientists are brought into the classroom on a regular basis is a specific example of this sort of activity. At present, there is no hard evidence to support a contention that the SEED program is of more value than other activities that could be purchased for equal amounts of money, but there seems to be little doubt that both the children involved and the mathematicians and scientists involved, are substantially benefited by the contacts.

Computer Activities in the Schools. In some respects, the most significant change that has occurred in the recent past and is occurring now in our society is in the use of computers. Virtually every business or government of any substantial size uses a computer in some way. Even if a person's vocation does not require him to become involved with computers, his dealings with governments (voting, paying taxes, etc.) and businesses will require that he have some contact with the omnipresent computer. Surely all children growing up in our society today should learn about the social implications of the computers (perhaps in the social science classes, perhaps in the mathematics classes), and futhermore, all children should have access to courses in computer mathematics, in the use of calculators, in automatic business data processing, and in key punching operations. Students at all academic levels can enhance their value in the vocational market place by being proficient in some aspects of the computer. This fact is attested to by the large numbers of commercial "schools" that have sprung up which promise to teach people some aspect of the computer business for a substantial amount of money. Surely, if public education means anything, such courses should be widely available in the public schools.

There are computer related courses available in New York State, and some children are taking such courses, however, it appears that only about 1% of the students in the secondary schools are actually taking computer related courses in a given year, so that less than 4% have such a course while they are in school. Futhermore, the majority of schools simply do not offer such courses. While the expenses are great for such courses, the benefits to the students and to society appear to be worth such expense. Time sharing computers with terminals in the school, distant computers to which students send punch cards, and small self contained computers in the schools are all possibilities. Both the economic and educational features of the various systems have been discussed at length in publications of various professional organizations. Whatever the procedure used, the State should have a plan by which computers can be made available in schools throughout the State.

In Table IV, the total numbers of computer related courses taught in the state of each type are listed.

TABLE IV

Numbers of computer related courses by types taught in New York State during the 1970-1971 academic year.

| Course description | Classes | Students | Teachers |
|---|---|---|---|
| Computer mathematics courses | 248 | 4,745 | 170 |
| Mathematics using calculators | 36 | 1,685 | 14 |
| Automatic Business Data Provessing I | 219 | 4,155 | 143 |
| Automatic Business Data Processing II | 57 | 1,083 | 44 |
| Key Punch Operations | 46 | 750 | 29 |

Content Modification. The major emphasis of the "new mathematics" movement of the past 15 years in the schools has been to change the curriculum to include mathematics of the past 300 years. Prior to 1957, most of the mathematics taught in our schools was material that was well known by mathematicians in the time of Newton. There has also been a tendency to teach many subjects earlier than they had previously been taught.

Some of the changes that have occurred in the name of the "new mathematics" are the following:

1. Earlier introduction of topics from algebra and geometry in the elementary schools (as mentioned earlier, this brings our curriculum closer to those of most European countries).
2. Substantially more teaching of probability and statistics in both the elementary and secondary schools.
3. Heavier reliance on graphs and graphing techniques.
4. Much more emphasis on inequalities.
5. Introduction of some work with computers (discussed earlier).
6. A new emphasis on mathematical structure.
7. Reduction in emphasis on computational techniques that may not be of as much use as before the advent of high speed computers and calculators.

In some situations there has also been great emphasis on certain kinds of vocabulary and great rigor in looking at the foundations of mathematics though there are many mathematicians and mathematics educators who feel that this emphasis may have been somewhat overdone in some instances.

From available information, it appears that the curriculum guides of New York State and New York City have more than kept pace with the changes at the national level. It would appear that New York is not behind the rest of the nation in these changes.

Need for Research. The promising practices discussed in this section have been used by good teachers in and out of New York State for many years. There are many other practices which have been tried and are being tried which hold promise of improving mathematics education. Those listed here are some of the more prominent practices that are not very widely used but have some evidence to support their use. Certainly it is true that a teacher or school system may be using none of these procedures and still be doing a good job of educating children within the existing situation, and a teacher or school system might appear to be following all of these recommendations and still be doing a very poor job of educating children.

Perhaps the most important thing that comes out of a review of the state of mathematics education is that nobody really knows very much for sure. Research evidence is sparse, and contradictory. Excellent practioners differ radically in practice and theory. Children learn (or fail to learn) in the most amazing ways. Undoubtedly, what is needed is more research. Not necessarily research to increase our knowledge about the learning process, but rather research that will encourage change for teachers and pupils. The Hawthorne Effect, mentioned at the beginning of this section may prevent educators from acquiring valid research evidence, but it can be the teacher's greatest ally. If people respond favorably whenever conditions are changed with their welfare in mind, then, teachers should be willing and anxious to change. Pupils would almost certainly respond favorably to most teacher instigated changes that apparently had the welfare of the pupils as their goal. If research tells us nothing else, it tells us this!

In order to encourage such research, schools and school systems should have personnel whose principal responsibility is to encourage research and help evaluate it. Unfortunately, available evidence shows that most people in the public schools who have research titles are primarily administrators and public relations men rather than researchers who are interested in improving the quality of education.

## Promising Practices Related to Teachers

The promising practices of the previous section pertained to activities that the teacher could carry out in a reasonably standard classroom. There have also been recent innovations in the use and education of the teachers themselves. Some of these will be discussed in this section.

Accountability and Performance Contracting. The history of education in this country is very different from the history of other professions such as law and medicine. While the other professions were allowed to function almost entirely under a free enterprise system until quite recently, education was thought to be so important to the welfare of the state that it became socialized. While it is true that private educational institutions have existed and made major contributions to education, even they were organized on a group basis, with one teacher teaching many students. Few individuals have been sufficiently wealthy to afford a private, individual, education for their children in the way in which they purchase private, individual medical and legal services.

Today, many people who believe that education is of utmost importance are asking whether the present system provides any assurance that children will actually receive a good education. The answer appears to be no. Clearly some children are receiving a second rate education. Clearly some teachers are doing a very poor job of educating children. What can be done to solve this problem? Make the teachers accountable for actually providing an excellent education.

Most people agree that ideally, teachers should be accountable for providing a quality education, just as an automobile manufacturer should be accountable for producing an automobile that will run safely. Such a statement raises many questions, however. To whom shall the teachers be accountable? For what shall we hold them accountable? How shall we decide whether they have achieved the agreed upon goals?

Any profession must, in the long run, be accountable to the society which it serves. However, the members of that profession ought, in general, to be accountable to the profession itself. Thus, if a physician or a lawyer behaves in an unprofessional manner, he may expect to be disciplined by his own colleagues. His colleagues are expected to evaluate his actions according to what was appropriate professional action in the situation involved. A physician is not punished when a patient dies, nor a lawyer when he loses a case, unless there is evidence that he did not follow accepted professional procedures. In a similar way, the education profession should create a procedure for evaluating the professional activities of its members, and taking appropriate action when the members of the profession do not carry out their responsibilities. At present, there is no widely accepted procedure of this sort in the profession, though the New York City school teachers and the New York City Board of Education are presently taking steps to institute such procedures.

The setting of goals in mathematics education is more complicated than is immediately apparent. Many students, parents, and teachers behave as though the goal of mathematics education is to get a higher score on the next mathematics test. In fact, upon further reflection, most people will agree that the true goals of mathematics education include helping the student to think quantitatively throughout the rest of his life; giving him the tools with which to do that thinking, or the means of acquiring those tools; encouraging creativity in mathematical activities; and fostering attititudes that will encourage children to use their mathematical abilities in appropriate ways throughout the rest of their lives. There are no known measuring instruments that can successfully determine whether a teacher of mathematics is achieving any of these goals. Tests are used to give some indication of the progress students are making towards some of the goals, but even the best tests give only a reasonable estimation of whether some of the goals are being met -- and then the measure can be relied upon only if the teacher is not educating explicitly for the test and the student not studying explicitly for the test. For if the goal becomes successful taking of a test, the long term goals will be achieved only incidently, if they are achieved at all.

Thus, standardized accountability appears not to be achievable in the immediate future, but professional accountability is feasible, and steps should be taken to achieve it as soon as possible.

Accountability has often been associated with performance contracting.

The above discussion of goals and testing makes clear the very real dangers inherent in any form of performance contracting, since the "performance" involved generally is performance on a test, not any long term goals. Any really good educator can raise scores on a specific test of a specific group of children, by a significant amount in any time period that exceeds sixty seconds; but any really good educator wouldn't claim that THIS is education.

Beyond the question of goals and measurement, there are many other questionable practices that have been associated with performance contracting in education. The question of extrinsic versus intrinsic motivation is an important one which has long bothered educators, but appears not to bother some performance contractors. Along with the question of motivation is the very real question of what happens to the children when they return to the regular classroom -- is the performance contractor responsible if after two years the child is in worse condition than he was at the beginning of the contract, or does the contractor simply get another contract to again temporarily raise the scores of the child through extrinsic motivation? The Hawthorne Effect, discussed earlier, is another factor that never seems to be considered when evaluating the results of a contractor's work.

There are many other problems inherent in contracts that have been written for educational performance in the past. These include the timing of the tests and subsequent extrapolation (virtually all children get lower test scores in October than they did the previous April, therefore, a contractor who guarantees to improve scores oneyear in the six months from October to April is virtually guaranteed of success even if the children make far less than average progress). The possibility that children will learn to deliberately get low scores on an early test so as to collect the rewards for improving. The fact that payments are often made to the contractor before he shows any results at all, and there appears to be no assurance that he will not actually harm the children.

Accountability in education is necessary and desirable. Before encouraging performance contracting in the state, it would be desirable to reexamine objectives and means of assessing attainment of those objectives and also to examine with great care the kinds of contracts that may be written. It is always fair to ask whether the managers of the present system couldn't sign the proposed contract and be assured of achieving results at least as good as those described in the contract.

Use of Professional Educators Other Than Classroom Teachers. One of the most obvious needs in education today is some reasonably objective method of deciding who is a good teacher and how other teachers can become better. Research on this subject in the teaching of mathematics provides no answers. Supervisors' ratings correlate well with each other, children's ratings correlate well with each other, colleagues' ratings correlate well with each other, but none of these variables correlates well with any of the other variables, and none of them correlate highly with performance of a teacher's pupils on tests. Mathematics educators, both in the elementary and secondary schools, and in the colleges and universities could make a substantial contribution to mathematics education by improving and validating objective techniques for evaluating mathematics teaching.

In an earlier section, diagnosis of pupil difficulties was discussed. Under present circumstances, classroom teachers have little time to carry out such diagnosis and often do not have the ability or training. Ideally, there should be a mathematics specialist for every eight to ten elementary school classrooms, who would be trained to diagnose difficulties and suggest appropriate remedial activities when indicated. Such a specialist would also be available to help teachers with more general problems in the teaching of mathematics including activities, mathematical concepts, evaluation, and curriculum. If it is deemed impossible to provide such specialists on such a broad basis, they could still be of considerable help if there were one specialist per school, or for two schools when the schools involved are reasonably small.

As discussed earlier, if such specialists did exist, there should be provisions for them to share problems and information with each other, and opportunities to use the presumed expertise of even higher level specialists in colleges and universities throughout the state.

Teacher Certification and Education. The deplorable state of teacher certification in mathematics in New York State has already been described. Clearly, a first step in improving mathematics education in New York is to require that mathematics teachers learn a reasonable amount of post high school mathematics. The Committee on the Undergraduate Program in Mathematics recommends a major in respectable mathematics with a minor in an area in which mathematics is used (for example,physics, mathematical economics, etc.). The major would include three semesters of elementary calculus plus approximately 30 semester hours of advanced work in abstract algebra, higher geometry, probability and statistics, computer mathematics, etc. While details might easily differ, it is clear that most mathematicians and mathematics educators would generally support the need for a teacher of high school mathematics to have a background approaching the recommended level.

At the elementary school level, it is generally believed that mathematics is the second most important subject taught (after the study of the English language). Surely the teachers of elementary schools should learn some mathematics in their college preparation to become teachers.

Beyond these immediate changes that should be made are many other possible modifications in teacher education and certification. Such teacher education activities as micro-teaching, video-taping, interaction analysis, role playing, etc., are discussed in detail elsewhere in the Commission's report, and therefore will not be considered here, except to point out that they can, with appropriate modifications, be very helpful in the education of mathematics teachers. Each of these procedures can be used to help future teachers (and in-service teachers) learn more about ways in which they can encourage mathematical thinking and creativity on the part of the children.

As teachers continue to develop a strong professional accountability, there should also be opportunities to strengthen the continuing education of mathematics teachers. This does not mean returning to college to "pick up" three academic credits from time to time. Rather, the teacher should sit down with a mathematics specialist in his school system from time to time and decide what his weaknesses appear to be and what could be done to remedy them. If the answer is to learn the content of a graduate or undergraduate course, that is what should be done (whether graduate credit is given or received is totally non-pertinent). If the best way for the teacher to acquire the necessary education is for him to take a job with one of the local industries for a summer, or a year, then, that should be possible. We must get away from the belief that official credit is synonymous with education.

If appropriate specialists were available, it should also be possible to have an on-the-job, long term, evaluation and education of a teacher before he attains permanent certification. This procedure should NOT be as automatic as it is at present.

Teacher Role. In the past, it has been common for teachers to spend as much as half of their time doing essentially non-professional activities such as collecting money, taking role, keeping records, baby sitting for children in a lunch room or study hall, typing and duplicating homework and tests, etc. While many teachers appear to be comfortable doing this kind of work, the education they have and the salaries they should receive are not commensurate with this sort of activity. Teachers should be given appropriate secretarial and para-professional help and then should be expected to do a full time job of educating children.

As professional standards rise and non-professional activities are carried out by other people, the teaching profession should become a higher prestige profession. Hopefully, salaries will follow the prestige, and people will attempt to enter the teaching profession with an enthusiasm equal to that for entering other professions. As we find out more about what makes an excellent teacher, it may also be possible to provide differential titles, salaries and duties to mark the true master teacher and encourage such master teachers to remain in teaching rather than entering administration. Additional duties might include the responsibility for helping with the education of future teachers in conjuction with a college or university.

## Recommendations

In so far as there is a problem in mathematics education in New York State, it is a problem affecting all the people of the state, and solving the problem is the responsibility of all the people of the state. Thus, the recommendations made here are not made only to professional educators nor are they made only to decision makers within the state government. There is something in these recommendations for virtually every citizen of the state. Many improvements can be made with little or no major outlays of financial resources, these are listed first. Some major improvements will come only at substantial cost to the taxpayers of New York, these are listed later.

Certification. As an immediate first step, the certification regulations should be changed back to those prevailing before the September 1969 rules became effective, so that high school teachers of mathematics would be required to have special methods course in the teaching of mathematics and somewhat more mathematics than their advanced high school pupils.

Once this step has been taken, a longer term study of the certification situation should be undertaken by a committee made up of State Education Department specialists, practicing mathematics and elementary school teachers, college mathematics educators and interested private citizens. This committee should be instructed to recommend certification procedures that consider not only professional education courses and content courses, but also the on-job quality of student-teacher relationships, and any other evidence that a candidate for certification is and will be an excellent teacher of mathematics.

Accountability. Professional teachers organizations ought to create goals for the teaching of mathematics and ought to provide machinery to see that progress is made towards achieving those goals. This should include specific objectives for teachers, suggestions for pre-service and in-service education, and committees to evaluate professional performance. It is very likely that research and evaluation specialists will be needed to help determine what is involved in being a good teacher, and how to measure it with reasonable objectivity. It would also be desirable for the professional organizations to involve citizen representatives in their deliberations.

If performance contracting is to be allowed in the state at some time in the future, a high level committee of educators and citizens ought to examine the experiences of schools in which such contracts have been written in the past, and suggest criteria for assuring that children are not seriously damaged and that appropriate evaluation procedures are used.

Regents Examinations. At present, Regents examinations are used as a predictor of college performance, as a status symbol for students and their parents, and as an accountability measure for teachers (it is widely assumed that a teacher is doing a good job if his pupils do well on Regents examinations). The Regents examinations also appear to improve minimal competency in college preparatory courses. The Regents Examinations should probably not be discontinued or replaced until a thorough review, at the philosophic level, is made by a citizens' committee to determine what is really needed in a testing program at the secondary school level. One possibility to be considered is an across the board minimal competency test similar to the PEP in the elementary school.

Innovation and Creativity. State Education Department personnel, local supervisors and administrators, and professional teachers' organizations should make a special effort to identify activities that appear to foster student creativity. These should be appropriately evaluated. News media should make a special effort to publicize such innovative efforts and the people responsible for them, as well as any objective evaluative information.

Non-Middle Class, Non-Standard-English-Speaking Children. In many respects, mathematics is by nature the most culture free of the subjects taught in the schools. Statistical achievement data support this contention. Thus, mathematics offers an ideal opportunity to reach children of differing cultural backgrounds. There is no objective evidence to suggest that children from different cultures (or female children) have less innate mathematical ability than Caucasian, middle class, English speaking males. If this be true, statistics demonstrate that our society is presently wasting most of its potential quantitative talent.

Curriculum directors, teachers, and others charged with planning and teaching the mathematics courses of the state should make a special effort to choose programs that are as culture free as possible, and as little oriented towards male supremacy as possible. In time, these spurious influences on the mathematics programs should be entirely eliminated with the possible exception of the language problem. The language problem can be partially eliminated by an approach that involves more physical activity on the part of the children, and by making available Spanish (or other) translations of the standard textbook (such translations are available for many American texts).

Modification of Content. State and local mathematics curriculum personnel should continue to monitor recommended subject matter changes and modifications. This has been well done in the past, and there is no reason to suppose that it will not be well done in the future.

At this time, there is no evidence to suggest that changing from any one of the commonly used mathematics programs to another one is particularly difficult for children. As greater modifications occur, this may become more of a problem. If so, the mathematics specialists recommended below w d be a great help in diagnosing and alleviating the difficulties.

Slow Learners. Classes in which slow learners are allowed to learn the mathematics of regular classes more slowly should be created. If the specialists recommended below are available, this should require no extra expense and will, according to research studies, produce substantially greater long term achievement. If the speicalists are not made avaliable, this recommendation should probably not be adopted, since transfer into and out of the program must be flexible (requiring substantial individual diagnosis and tutoring), and original diagnosis is difficult.

The phrase "slow learner" here is being used to mean precisely what it says--one who learns (mathematics ) slowly. It is NOT a synonym for inner city youth, or rural youth, or culturally disadvataged (what ever that means) youth, or troubles makers. The identification of children who learn mathematics slowly, as opposed to childre who are having troubles in school because of various problems that society has created for itself, is sometimes difficult, but it is important that the distinction be made if a program for slo learners is going to work.

Involving Parents. Parents and other adult members of the community ought to take an active interest in the education of children. In particular, in mathematics, it would be desirable for teachers to provide a short description of what the children will be learning over the next several weeks, and what activities parents or other members of the household could carry on with the children to improve their acquisition of the skills and knowledge to be learned. Ideally, textbook authors might provide such letters for each chapter or unit of their material, and teachers could make appropriate modifications before sending the letters out to parents. Of course, such a program will work only if parents do take an active interest, but such letters would encourage parents who do not have time or facilities to get such information on their own to take an active interest.

Use of Local Resources. Most adults use mathematics in one way or another (or wish they could use it, and would be better off if they did). Some of these adults would be willing and able to talk with students about the ways in which mathematics can be used, and some would be able to teach classes about ways in which they use mathematics. If people who were willing to volunteer for such service were once identified, and their abilities catalogued, the process of evaluating subsequent experiences with them and keeping the records up to date would not be a difficult job. Then, teachers who wanted somebody to speak, for example, on the use of mathematics in auto-mechanics, could ask the central cataloguing person for a few names and telephone numbers, and could then bring this resource to their pupils. The initial expense would be approximately the salary of one teacher per school for one summer, plus mailing expenses and a secretary. The expense could be borne by local schools or by the state.

Information about the desirability of mathematics being learned as a practical discipline, and information regarding the advantages of exposing students to different situations from time to time, both support the desirability of a plan such as this.

Activity Approach to Mathematics. Curriculum planners and teachers should try to incorporate appropriate activities into the mathematics curriculum. Ideally, textbooks should be written with appropriate activities built into the children's textbooks so that they are forced out of the textbook and into the real world from time to time. The activities must be integrated with the mathematics program, rather than totally separate from it in order to gain maximum benefit.

Teachers should have money available to buy inexpensive materials for such activities. They might, for example wish to purchase construction card board, wood, nails, rubber bands, popsicle sticks, or other inexpensive materials which can be used to improve the teaching of mathematics. Probably about $100 per teacher the first year and $50 per teacher for subsequent years would be a great plenty. Although elementary school teachers teach mathematics to only one class, their pupils are far more likely to be benefited by use of such materials, and therefore it is not unrealistic to assume that elementary school teachers and secondary school teachers could profitably spend about the same amount.

The total possible cost of such a program would be about $7,500,000 for the first year and $3,750,000 for the subsequent year. In fact, however, it is likely that many teachers would not use their full quota, and the actual expense could easily be half of these amounts.

Computers. Every child in the State of New York should have access courses pertaining to computers at various levels. In order to be aningful, such courses must make actual computers available to the students, en if only by mail. The most economical and educationally effective means achieving this goal is not immediately obvious. Several professional groups ve studied the situation and made recommendations. The State Education thematics Specialists should be directed to make a study of the resources d requirements for computers in New York State, and the changes that are kely to occur in the next five to ten years, and then make a recommendation. e cost of such a study would be minimal, involving several months of the ecialists' time and less than 100 days of consultants' time. The ultimate st of the program itself is likely to be very substantial, but the benefits the people of New York are made obvious by the large number of "schools" at have been created in this state and others which purport to teach people e computer business.

State Education Department Specialists. Many of the recommendations of is report have indicated the desirability of some substantial activity on e part of the S.E.D. specialists. It is likely that if substantial numbers f these recommendations are adopted, the number of specialists in mathematics hould be increased, and some assignments modified. This question should be eviewed by the specialists in concert with the appropriate officials in the tate Government once a decision has been made regarding these recommendations.

Specialists. Throughout this report, a case has been made for the need for more specialists in mathematics. The principal duties of such specialists would be to have in-depth contact with individual students to diagnose the difficulties the students are having, and plan courses of action to allaviate those difficulties. The specialists would also be expected to work closely with regular classroom teachers in planning lessons and other activities.

The specialists would not have primarily administrative or supervisory duties. Provision should be made for specialists to get together with other specialists at regular intervals to share problems and promising practices. They should also have available to them more advanced specialists who presumably would have more experience and knowledge (probably professors in nearby teachers colleges or schools of education).

Specialists should not have permanent certification, but rather, their performance should be reviewed from time to time by a state wide board. This Board would review a specialist's performance in light of available information on the children, teachers, and others involved. The Board could place a specialist on provisional certification at any time, with a specific length of time in which the speicalist would be expected to achieve specific results or lose his certification as a specialist. Presumably, the specialist would retain permanent certification as a teacher.

A specialist in elementary mathematics should be a certified elementary school teacher with some experience (at least two years) teaching in elementary schools. He should have a substantial mathematics background, including number theory, abstract and linear algebra, geometry, probability and statistics, calculus and applications of mathematics to several fields. He should also have a substantial background in child development, and should have at least a one year course in the teaching of elementary school mathematics. The specialist also ought to participate in a full time internship in diagnosing and prescribing for children's mathematics learning problems, in conjunction with a seminar on the same subject.

At the secondary level, a specialist should have essentially the same sort of mathematics background as a regular teacher, but he should also have special work in developmental psychology, and tests and measurement.

He should be a certified secondary school teacher of mathematics with at least two years of teaching experience. Such a specialist also ought to have participated in a full time internship, in conjunction with a seminar, in diagnosing and prescribing for children's learning problems in mathematics.

At the elementary school level, a specialist should be able to properly service between 200 and 300 children. A larger load than this could be expected to result in a drastic reduction in effectiveness, since the specialist could not get to know many more than 300 individuals well enough to really understand their problems. This means that when the program is fully staffed, approximately 6,000 elementary school mathematics specialists would be needed. At approximately $10,000 per specialist per year, this comes to $60,000,000 a year. At the secondary level, because of the greater specialization of the teachers and the expectation that there should be fewer problems (both because of a successful elementary program and the self selection nature of many of the high school mathematics courses) approximately 2,000 specialists should be sufficient, for a total cost of about $20,000,000 per year. Thus, the full program of specialists in mathematics could be expected to add about $5 per year to the taxes of an average New York resident, and could be expected to substantially increase the quantitative competence of the state's residents.

Because of the great expense involved, the somewhat untried nature of the program, and the total unavailability of well trained, available specialists, there would be substantial benefits to trying the program on a small scale for several years. In such a trial program, about 100 specialists could be trained for one year and would then work in the schools in which they served their internship for another two years. The entire program would be evaluated in terms of the effects on the children involved in the program. Because of the uncertain nature of the program from the point of view of the potential specialists, and the desirability of having outstanding people apply for such positions, it would be desirable to pay the potential specialists during their internships and also to pay for the cost of their supervision and seminars. Thus, the cost of this trial program would be about $1,000,000 per year for three years.